

Im Oberseminar

## Deformationsquantisierung

spricht am **16.10.2015 um 14 Uhr c.t.**,

im Seminarraum 00.009 (Physik Ost)

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über das Thema:

### Duality of uniform spaces and bornological unital abelian partial $*$ -algebras

By the Gel'fand-Naimark theorem, the category of abelian  $C^*$ -algebras is equivalent to the category of compact Hausdorff spaces via two functors  $\Phi$  and  $\mathcal{A}$  that assign to every abelian  $C^*$ -algebra  $A$  the topological space  $\Phi(A)$  of its characters with the weak- $*$ -topology and to every compact Hausdorff space  $X$  the  $C^*$ -algebra  $\mathcal{A}(X)$  of continuous complex-valued functions over  $X$ . On the arrow-side, they simply map every continuous unital  $*$ -homomorphism / every continuous function to its pull-back.

This theorem lies at the heart of non-commutative geometry and strict deformation quantisation in the spirit of Rieffel's work, where abelian  $C^*$ -algebras correspond to ordinary commutative geometry and classical observable algebras and non-abelian  $C^*$ -algebras to non-commutative geometry and quantum observable algebras. If one tries to generalise this to the realm of e.g. Fréchet- $*$ -algebras, then one also has to generalise the Gel'fand-Naimark theorem:

It is immediately clear that the construction of the space of characters of a general unital abelian  $*$ -algebras and of the  $*$ -algebra of complex-valued continuous functions over a general topological space still make sense, but they do not establish an equivalence of these two categories. Nevertheless, one can ask what the largest sub-categories of unital abelian  $*$ -algebras and topological spaces are, such that this construction induces an equivalence of categories. This question was basically solved with the introduction of realcompact spaces by Edwin Hewitt.

I suggest that this duality can be generalised even more by transferring the construction to uniform spaces and the partial  $*$ -algebras of the uniformly continuous complex-valued functions over them. In this talk, I will give the definition of uniform spaces and (bornological) unital abelian partial  $*$ -algebras, discuss their basic properties and explain the duality between these two categories.

gez. Stefan Waldmann